H+D

Controls of eolian dune size and spacing

N. Lancaster
Department of Geology, Arizona State University
Tempe, Arizona 85287-1404

ABSTRACT

Data for the Namib and Gran Desierto sand seas suggest that the controls of dune size and spacing are complex. The relation between dune height and spacing varies with dune type and location and reflects both dune dynamics (vertical accretion vs. migration or extension) and availability of sand. There is no general relation between dune spacing and grain size. In the Namib sand sea the height of compound and complex dunes (draas) is inversely proportional to potential sand-transport rates, whereas the height of dunes superimposed on their flanks varies directly with potential sand-transport rates. These observations can be combined with data on dune spacing to demonstrate the existence of a hierarchy of eolian dunes, each element of which responds to variations in sand-transport rates at different temporal and spatial scales. Whereas the morphology of individual simple dunes and superimposed dunes on draas is related to contemporary rates and directions of sand transport, the morphology and development of draas reflects long-term and regional patterns of sand transport and deposition.

INTRODUCTION

Desert dunes occur in a variety of morphodynamic types, each of which displays a range of size (height, width) and spacing. The occurrence of each major dune type (crescentic, linear, or star) is largely controlled by the directional variability of the wind regime. Wind speed, grain size, sand supply, and vegetation cover play subordinate roles (Fryberger, 1979; Lancaster, 1983; Wasson and Hyde, 1983a). In comparison, the factors that control the size and spacing of eolian dunes are poorly understood. Hanna (1969), Wilson (1972), Folk (1976), and Brown (1983) have argued that dune size and spacing are controlled by regular transverse or longitudinal vortices developed in the atmospheric boundary layer. However, field studies of wind flow over dunes, such as those by Tsoar (1983) and Livingstone (1986), cast doubts on the existence of such vortices. Wilson (1972) further suggested that the size of the coarse fraction of dune sands sets the scale of the turbulent atmospheric structures that determine dune spacing, indicating a grain size control of dune spacing.

Data from studies of dune morphometry and processes in sand seas and dune fields in the Namib and Kalahari deserts of southern Africa and the Gran Desierto of northern Mexico provide insights into the nature of the controls of dune size and spacing. Terminology used here for dune types and varieties follows McKee (1979). Compound dunes (superimposition of dunes of the same morphodynamic type) and complex dunes (superimposition of dunes of different types) are referred to collectively as draas, following Kocurek (1981).

NATURE OF DUNE PATTERNS

Satellite images and aerial photographs of desert sand seas show that most dune patterns are regular. Data on dune height and crest-to-crest spacing for crescentic, linear, and star dunes of simple, compound, and complex varieties, compiled from my field measurements, together with information from Wasson and Hyde (1983b) for Australian dune fields, show a good correlation between dune height and spacing (Fig. 1a). There are similarly close correlations between dune width and spacing (Breed and Grow, 1979; Lancaster, 1983; Wasson and Hyde, 1983a, 1983b).

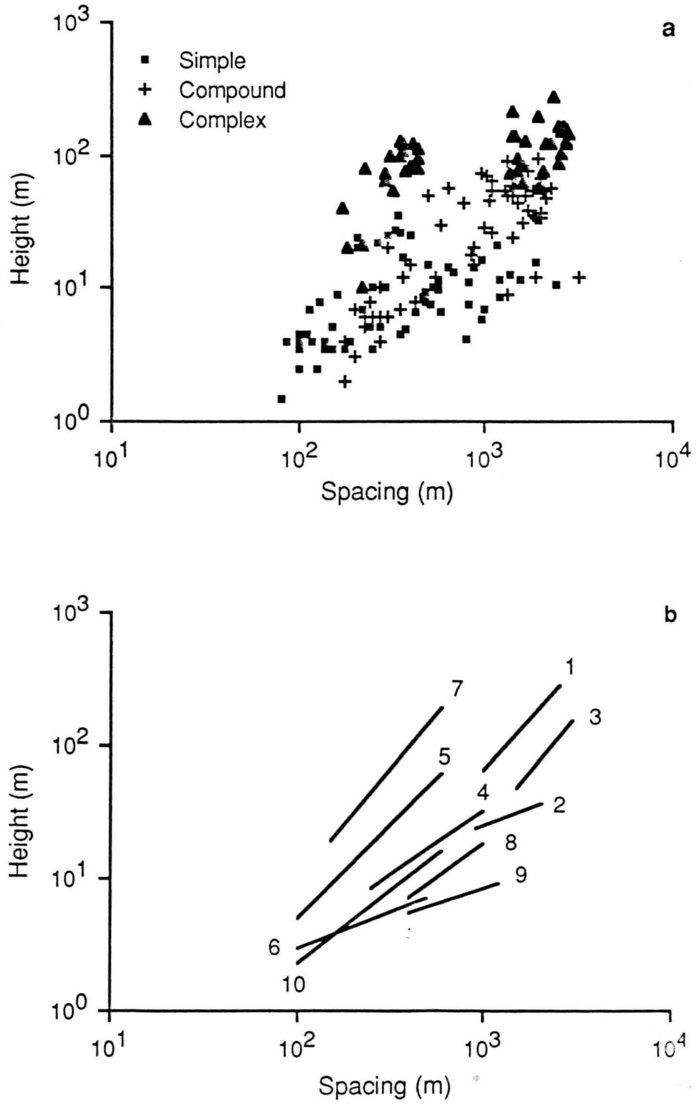

Figure 1. Relation between height and spacing of eolian dunes. a: For all dune types (r = 0.80). Note overlap in height and spacing between simple, compound, and complex varieties. b: For different dune types and localities; 1—Namib sand sea, star dunes; 2—Namib sand sea, compound linear dunes; 3—Namib sand sea, complex linear dunes; 4—Namib sand sea, crescentic dunes; 5—Skeleton Coast dune field, crescentic dunes; 6—Gran Desierto, crescentic dunes; 7—Gran Desierto, star dunes; 8—Simpson-Strzelecki, linear dunes; 9—Great Sandy Desert, linear dunes; 10—southwestern Kalahari, linear dunes. See also Table 1.

suggest that this fraction may be largely inherited or xenocrystic; therefore, this fraction was not used for the intrusion age interpretation.

DISCUSSION

The nature and age of the Station Creek Formation–Barnard Glacier pluton–Kaskawulsh metamorphic rock contacts are critical elements of the late Paleozoic tectonic evolution of Wrangellia and the Alexander terrane. We present unequivocal evidence that the Station Creek Formation and the Kaskawulsh metamorphic rocks were stitched together at 309 ±5 Ma by the Barnard Glacier pluton. Therefore, the Alexander terrane and Wrangellia were contiguous during the Middle Pennsylvanian. The exact nature of the Station Creek Formation–Kaskawulsh metamorphic rock contact has not been resolved, although two possibilities seem most likely with the available data. The Station Creek Formation and the Kaskawulsh metamorphic rocks may have been juxtaposed by major faults prior to the Middle Pennsylvanian. Alternatively, the Station Creek Formation may have been deposited unconformably on the Kaskawulsh metamorphic rocks. The hypotheses can be tested by further detailed mapping of the contact relations.

The fact that Wrangellia and the Alexander terrane were contiguous during the Middle Pennsylvanian provides an important new constraint on paleogeographic reconstructions of the northwest Cordillera. Widely accepted interpretations of the paleogeography maintain that the terranes evolved separately until the Jurassic or later, on the basis of differences in Paleozoic and Mesozoic stratigraphy (Jones et al., 1977) and paleomagnetic data (Panuska and Stone, 1985; Hillhouse, 1987). These interpretations are no longer tenable in light of the conclusions of this study. Unquestionably, disparate stratigraphic relations at some locations indicate that parts of the terranes were displaced by faults; however, these relations do not require that the terranes evolved separately. Only limited paleomagnetic data are available for the late Paleozoic and early Mesozoic age rocks of the terranes (Panuska and Stone, 1985), and these data are imprecise with respect to the relative paleogeographic positions of the terranes. The stratigraphic and paleomagnetic data must be reevaluated, and new paleogeographic models are needed. The new data presented here and recent studies of the Triassic volcanic sequences of Wrangellia and the Alexander terrane (Davis and Plafker, 1985; MacIntyre, 1986) suggest that the terranes evolved together for significant parts of the late Paleozoic and early Mesozoic. The speculative possibility that the terranes evolved together throughout their respective geologic histories, as suggested by Muller (1977), needs to be reconsidered. Several interpretations are permissible with the available data; however, any interpretation must account for Middle Pennsylvanian juxtaposition of the terranes.

Pennsylvanian-Permian plutonic rocks are widespread in eastern Alaska (Richter et al., 1975; MacKevett, 1978; Barker and Stern, 1986; Aleinikoff et al., 1988), southwestern Yukon, and southeastern Alaska (Hudson et al., 1977; Campbell and Dodds, 1982a, 1982b; Dodds and Campbell, 1988), and are petrologically and geochemically similar to the Barnard Glacier pluton (S. C. Bergman, in prep.). The fact that the Barnard Glacier pluton stitched together the basement assemblages of Wrangellia and the Alexander terrane provides strong evidence that Pennsylvanian-Permian plutonic rocks of the northwest Cordillera are a continuous belt of genetically related rocks.

REFERENCES CITED

Aleinikoff, J.N., Plafker, G., and Nokleberg, W.J., 1988, Middle Pennsylvanian plutonic rocks along the southern margin of Wrangellia, *in* Hamilton, T.D., and Galloway, J.P., eds., U.S. Geological Survey in Alaska: Accomplishments during 1987: U.S. Geological Survey Circular 1016, p. 110–113.

Barker, F., and Stern, T.W., 1986, An arc-root complex of Wrangellia, E. Alaska Range: Geological Society of America Abstracts with Programs, v. 18, p. 534.

Berg, H.C., Jones, D.L., and Richter, D.H., 1972, Gravina-Nutzotin belt—Tectonic significance of an upper Mesozoic sedimentary and volcanic sequence in southern and southeastern Alaska: U.S. Geological Survey Professional Paper 800-D, p. D1–D24.

Campbell, R.B., and Dodds, C.J., 1982a, Geology of s.w. Kluane Lake map area, Yukon Territory: Geological Survey of Canada Maps 115F and 115G, Open File 829, scale 1:250,000.

—— 1982b, Geology of the Mount St. Elias map area, Yukon Territory: Geological Survey of Canada Maps 115B and 115C, Open File 830, scale 1:250,000.

Coney, P.J., Jones, D.L., and Monger, J.W.H., 1980, Cordilleran suspect terranes: Nature, v. 288, p. 329–333.

Davis, A., and Plafker, G., 1985, Comparative geochemistry and petrology of Triassic basaltic rocks from the Taku terrane on the Chilkat Peninsula and Wrangellia: Canadian Journal of Earth Sciences, v. 22, p. 183–194.

Dodds, C.J., and Campbell, R.B., 1988, Potassium-argon ages of mainly intrusive rocks in the St. Elias Mountains, Yukon and British Columbia: Geological Survey of Canada Paper 87-16, 43 p.

Gardner, M.C., MacKevett, E.M., and McClelland, W.D., 1986, The Chitina fault system of southern Alaska—An Early Cretaceous collisional suture zone: Geological Society of America Abstracts with Programs, v. 18, p. 108.

Gehrels, G.E., and Saleeby, J.B., 1987, Geological framework, tectonic evolution, and displacement history of the Alexander terrane: Tectonics, v. 6, p. 151–173.

Hillhouse, J.W., 1987, Accretion of southern Alaska: Tectonophysics, v. 139, p. 107–122.

Hudson, T., Plafker, G., and Lanphere, M.A., 1977, Intrusive rocks of the Yakutat–St. Elias area, south-central Alaska: U.S. Geological Survey Journal of Research, v. 5, p. 155–172.

Jones, D.L., Silberling, N.J., and Hillhouse, J.W., 1977, Wrangellia—A displaced terrane in northwestern North America: Canadian Journal of Earth Sciences, v. 14, p. 2565–2577.

Jones, D.L., Silberling, N.J., Coney, P.J., and Plafker, G., 1987, Lithotectonic terrane map of Alaska (west of the 141st meridian): U.S. Geological Survey Miscellaneous Field Studies Map Mf-1874-A, scale 1:2,500,000.

MacIntyre, D.G., 1986, The geochemistry of basalts hosting massive sulfide deposits, Alexander terrane, northwest British Columbia: British Columbia Ministry of Energy, Mines and Petroleum Resources Paper 1986-1, p. 197–210.

MacKevett, E.M., Jr., 1978, Geologic map of the McCarthy quadrangle, Alaska: U.S. Geological Survey Miscellaneous Investigation Series Map I-1032, scale 1:250,000.

MacKevett, E.M., Jr., and Jones, D.L., 1975, Relationships between Alexander and Taku-Skolai terranes in the McCarthy quadrangle: U.S. Geological Survey Professional Paper 975, p. 69.

MacKevett, E.M., Jr., Hudson, T., and Tysdal, R.G., 1975, Relationships between the Taku-Skolai and Alexander terranes in the McCarthy quadrangle, *in* Carter, Claire, ed., U.S. Geological Survey Alaska Program, 1974: U.S. Geological Survey Circular 700, p. 51.

MacKevett, E.M., Jr., Gardner, M.C., Bergman, S.C., Cushing, G.W., and McClelland, W.D., 1986, Geological evidence for Late Pennsylvanian juxtaposition of Wrangellia and the Alexander terrane, Alaska: Geological Society of America Abstracts with Programs, v. 18, p. 128.

Muller, J.E., 1967, Kluane Lake map-area, Yukon Territory: Geological Survey of Canada Memoir 340, 137 p.

—— 1977, Evolution of the Pacific margin, Vancouver Island, and adjacent regions: Canadian Journal of Earth Sciences, v. 14, p. 2062–2085.

Panuska, B.C., and Stone, D.B., 1985, Latitudinal motion of Wrangellia and Alexander terranes and the southern Alaska superterrane, *in* Howell, D.G., ed., Tectonostratigraphic terranes of the Circum-Pacific region: Circum-Pacific Council for Energy and Mineral Resources, Earth Science Series No. 1, p. 109–121.

Richter, D.H., Lanphere, M.A., and Matson, N.A., 1975, Granitic plutonism and metamorphism, eastern Alaska Range, Alaska: Geological Society of America Bulletin, v. 86, p. 819–829.

ACKNOWLEDGMENTS

We thank Travis Hudson, Kris Meisling, Dave Doherty, Peter Coney, George Gehrels, and Karl Karlstrom for helpful suggestions on this manuscript. We thank ARCO Alaska and ARCO Oil and Gas Company for permission to use company data in this paper.

Manuscript received February 4, 1988
Revised manuscript received June 23, 1988
Manuscript accepted June 30, 1988

Much of the scatter of data points in Figure 1a arises because the tion between dune height (D_H) and spacing (D_S) varies with dune type location (Fig. 1b). The relation can be represented by a power func- ı (Lancaster, 1988),

$$D_H = c\,D_S^{\,n}, \tag{1}$$

exponents of which vary between 0.52 and 1.72, from one sand sea to another, as well as from one dune type to another in the same sand sea (Table 1). An exponent of unity indicates that dune height increases at the same rate relative to dune spacing. A given amount of sand can be formed into a few widely spaced dunes or many small, closely spaced dunes. Where the exponent is greater than unity (e.g., complex linear dunes and star dunes in the Namib and Gran Desierto sand seas), dune height increases more rapidly than dune spacing, indicating a tendency for vertical growth of the dunes. This may reflect an abundant supply of sand and a wind regime that promotes deposition on the dunes. Exponents less than unity indicate that dune height increases less rapidly than dune spacing. In these examples (compound linear dunes in the Namib sand sea, linear dunes in the Great Sandy Desert, and crescentic dunes in the Gran Desierto), dune size may be limited by the availability of sand. The relation between dune height and spacing therefore appears to reflect both the availability of sand for dune building and wind regime characteristics, which determine whether dunes will tend to accrete vertically (star dunes and complex dune varieties), migrate (simple crescentic dunes), or extend (many simple and compound linear dunes).

In many sand seas and dune fields, large dunes are characterized by the development of superimposed smaller dunes on their stoss or lee slopes (compound or complex dunes in the terminology of McKee, 1979). In areas of these dunes, it is possible to identify a clear hierarchy of dune spacing with two elements that correspond to the main dunes (or draas) and the smaller dunes that are superimposed upon them (Fig. 2). The mean size of each variety is different (Mann-Whitney U test, significant at the 0.05 level), suggesting that a minimum primary dune size must be reached before superimposed dunes can develop.

TABLE 1. RELATION BETWEEN DUNE HEIGHT AND SPACING

Locality	Power function exponent	Correlation coefficient
amib sand sea		
Crescentic	0.97	0.83
Compound linear	0.54	0.66
Complex linear	1.72	0.72
Star	1.20	0.62
keleton Coast dune field - crescentic	1.20	0.95
ran Desierto		
Crescentic	0.58	0.76
Star	1.70	0.75
Simpson-Strzelecki dune field - linear	1.06	0.67
Great Sandy Desert - linear	0.52	0.70
Southwestern Kalahari - linear	1.10	0.81

Note: Data from Figure 1b, this paper.

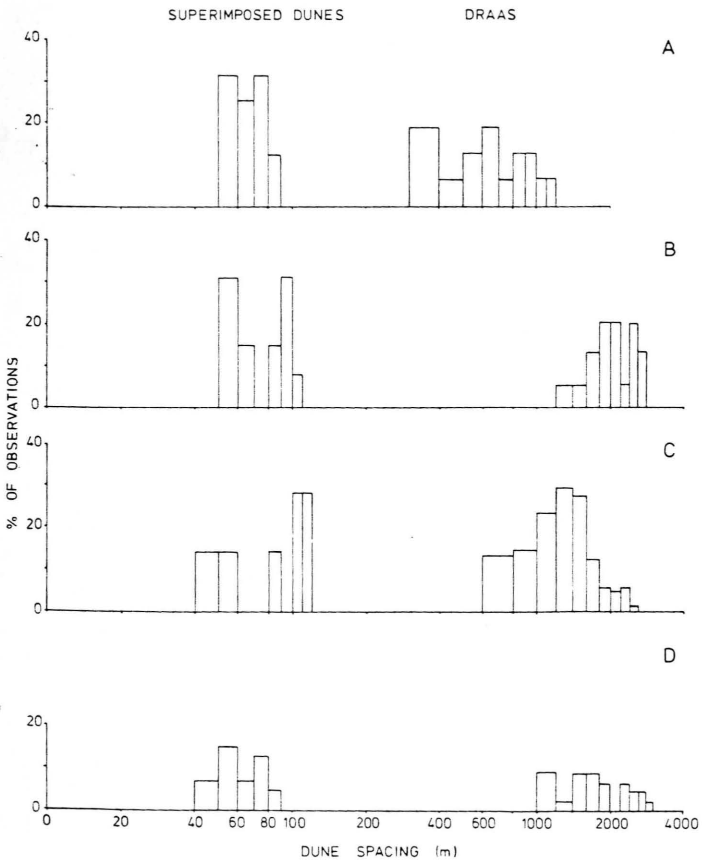

Figure 2. Hierarchical arrangement of dune and draa spacing in Namib and Gran Desierto sand seas. A: Namib sand sea, compound crescentic; B: Namib sand sea, compound and complex linear; C: Namib, star; D: Gran Desierto, compound crescentic.

INFLUENCE OF GRAIN SIZE ON DUNE SPACING

Wilson (1972) found a clear relation between bed-form spacing and the grain size of the coarse 20th percentile of dune-crest sands (P_{20}) in three northern Saharan sand seas. However, data for dune fields and sand seas in the Namib, the southwestern Kalahari, and the Gran Desierto of Mexico (Fig. 3) show that there is no general relation between dune spacing and grain size in these areas. Similarly, recent data from Australia (Wasson and Hyde, 1983b) show no relation between P_{20} and linear dune spacing. These data suggest that Wilson's hypothesis of a grain size control of dune spacing is not generally applicable. In a given sand sea, the spacing of some dune types is apparently correlated with P_{20}, whereas other types show no relation. The spacing of complex and star dunes in the Namib and

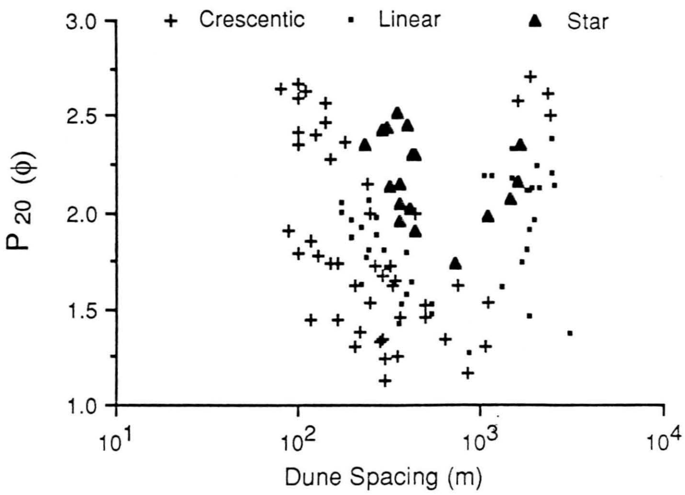

Figure 3. Relation between dune spacing and coarse 20th percentile of dune sands (P_{20}). Data from Namibian, Kalahari, and Gran Desierto sand seas and dune fields.

Gran Desierto sand seas is unrelated to P_{20}. However, data for simple and compound crescentic dunes show that their spacing tends to increase with a coarsening of P_{20}. A possible explanation for this relation is to be found in studies of winds and sand transport rates over dunes (e.g., Lancaster, 1985a; Tsoar, 1986) which indicate that differences in sand-transport rates between the base and crest of the stoss slope result in an increase in crescentic dune length (measured parallel to the wind) and crest-to-crest spacing with an increase in grain size.

INFLUENCE OF SAND-TRANSPORT RATES

Several workers have suggested that dune size and spacing increase as wind speed and sand-transport rates increase. Wilson (1972) suggested that dune spacing varied approximately with the cube of wind speed. Glennie (1970) and Besler (1980) argued that large compound and complex dunes were formed during periods of strong winds in Pleistocene glacial periods, and that the small dunes which are superimposed on their flanks are the products of weaker modern winds.

By analogy with subaqueous bed forms (Allen, 1968; Rubin and Hunter, 1982), it might be expected that dune size is related to mean sand-transport rates, so that larger dunes are located where sand-transport rates are high, and small dunes occur where transport rates are low. However, for 100 km^2 areas of dunes in the Namib sand sea the opposite situation occurs: large dunes are found in areas where annual potential sand-transport rates (calculated using the formula of Bagnold, 1953) are low, and small dunes occur in areas of high potential sand-transport rates (Fig. 4a). Information on dune size in Wilson (1973), Breed and Grow (1979), and Breed et al. (1979) also suggests that in Saharan sand seas large dunes occur where annual total or net potential sand transport is low.

However, when individual complex dunes in the Namib sand sea are considered, there is a relation between the spacing and height of the superimposed crescentic dunes and the measured wind speed and potential sand-transport rates at different points on the dune (Fig. 4b). The height of superimposed dunes becomes smaller as wind velocities and potential sand-transport rates decrease on the lee side of complex linear dunes, as also observed by Rubin and Hunter (1982). These observations suggest that superimposed dunes and draas are formed by two different scales of air-flow variability and can coexist in the same flow in a manner similar to that of the subaqueous bed forms studied by Boothroyd and Hubbard (1975), Smith and McLean (1977), and Rubin and McCullough (1980). The morphology of superimposed dunes on draas is controlled by secondary air flows created by the draas, whereas the draas are formed by the primary air flow. Their size appears to be the result of long-continued growth in wind regimes that give rise to deposition on the dune, and their occurrence in a sand sea reflects regional sand-transport patterns in which sand is transported from areas of high transport rates and deposited in areas of low or decreasing transport rates (Lancaster, 1988).

DISCUSSION

In the sand seas and dune fields studied by me and by Wasson and Hyde (1983a, 1983b), dunes of different types appear to develop in different ways, and there are no generally applicable relations between dune size or spacing and grain size or wind-regime characteristics. The relations between dune size and potential sand-transport rates indicate that the size of superimposed dunes increases with transport rates, whereas draa size decreases with transport rates. Therefore, it appears that the factors that control dune size and spacing are determined in part by dune type and dune size.

First, this is a result of the nature of dune dynamics. As the dune grows, it projects into the boundary layer and creates secondary patterns of air flow on and around itself. These secondary flows have been shown to play a major role in the development of dunes by their control of patterns of erosion and deposition and the dynamics of superimposed dunes on draas (Tsoar, 1983; Lancaster, 1985a; Livingstone, 1986; Havholm and Kocurek, 1988).

Second, there is a hierarchical system of eolian dunes which consists of (1) individual simple dunes or superimposed dunes on draas, and (2) draas. There is a characteristic time period, termed the relaxation or reconstitution time (Allen, 1974), over which each element of the hierarchy will adjust to changed conditions. Because change in dunes involves movement of sediment, an increasing spatial scale is therefore involved at each level of the hierarchy.

The reconstitution time can be represented by the time taken for the bed form to migrate one wavelength in the direction of net transport. In the Namib sand sea, typical complex linear dunes have a spacing of 2100 m and migrate at a rate of 0.05 m/yr. The reconstitution time for these

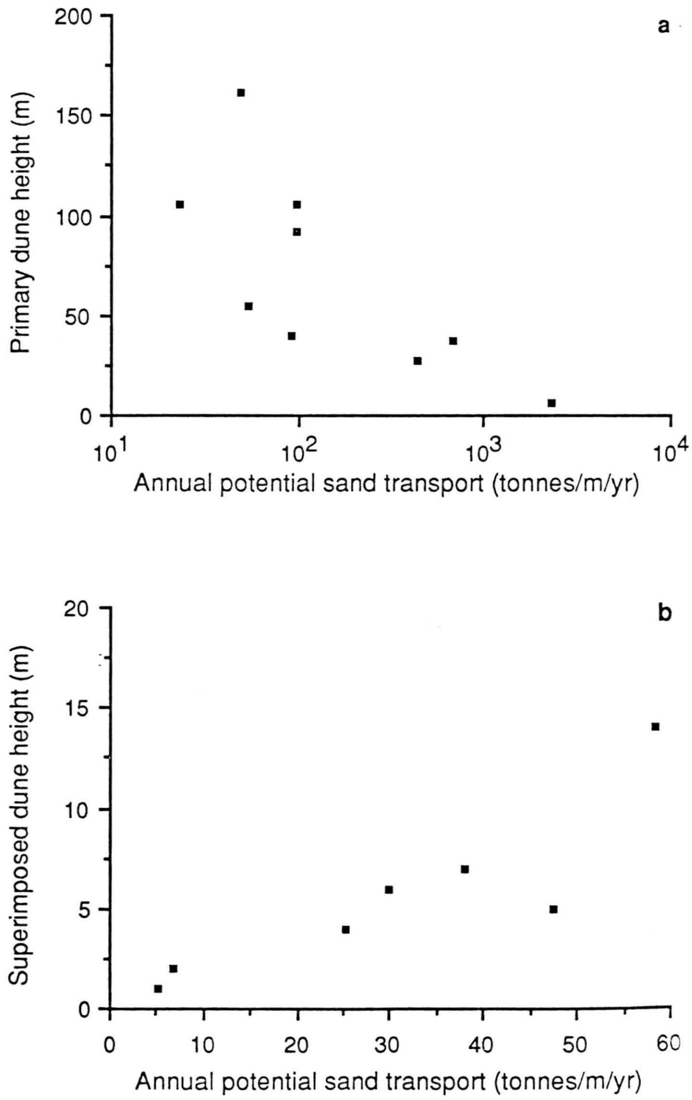

Figure 4. a: Relation between mean dune height and annual total potential sand-transport rate in Namib sand sea. Data on potential sand transport rates from Lancaster (1985b). b: Relation between mean height of crescentic dunes superimposed on east flank of complex linear dunes in Namib sand sea and potential sand-transport rates calculated from wind-speed measurements at crest of each superimposed dune.

ines is therefore 42 ka. Crescentic dunes superimposed on the flanks of the linear dunes have a mean spacing of 90 m, and they migrate at a rate of m/yr, giving a reconstitution time of 30 yr. Reconstitution time therefore increases by several orders of magnitude from simple dunes to draas. This implies that the morphology of simple dunes and superimposed dunes is governed principally by annual or seasonal patterns of wind speed and direction and by spatial changes in wind speed over draas. The life span of these dunes is about 10 to 100 yr. Draas are relatively insensitive to seasonal changes in local air flow conditions and may persist for 1 to 100 ka. Their size is not a direct function of grain size or sand-transport rates, but is the result of long-continued growth in conditions of abundant sand supply. The distribution of draas in sand seas is controlled by regional-scale patterns of winds and sand-transport rates.

The Algodones dunes, California, are an excellent example of these principles. The crescentic draas are approximately transverse to the mean annual resultant sand-transport direction, whereas the superimposed crescentic dunes change their orientation in response to seasonal changes in wind direction (Havholm and Kocurek, 1988).

CONCLUSIONS

The differences in scale between the elements of the eolian bed-form hierarchy suggest that the factors which control the size and spacing of simple dunes should be considered separately from those that influence the size and spacing of draas. Variations in winds and sand-transport rates at different temporal and spatial scales appear to be the most important control of eolian dune size and spacing. Whereas the height and spacing of individual simple and superimposed dunes in active sand seas probably tend toward an equilibrium with respect to contemporary sand-transport rates and directions, the size and spacing of draas are functions of the long-term pattern of accumulation of sand in certain areas of the sand sea, determined by regional-scale patterns of winds and sand-transport rates.

Although the data on dune morphometry and dynamics are limited, they do suggest that the size and spacing of eolian dunes cannot be explained by means of simple relationships. Interpretations of both modern dune patterns and ancient eolian sandstones must take account of this.

REFERENCES CITED

Allen, J.R.L., 1968, Current ripples: Amsterdam, Elsevier, 433 p.

——1974, Reaction, relaxation and lag in natural sedimentary systems: General principles, examples and lessons: Earth Science Reviews, v. 10, p. 263–342.

Bagnold, R.A., 1953, Forme des dunes de sable et régime des vents: Actions éoliennes: Centre National de Recherches Scientifiques, Colloques Internationaux, v. 35, p. 23–32.

Besler, H., 1980, Die Dunen-Namib: Entstehung und dynamik eines ergs: Stuttgarter Geographische Studien, v. 96, 214 p.

Boothroyd, J.C., and Hubbard, D.K., 1975, Genesis of bed forms in mesotidal estuaries, *in* Cronin, L.E., ed., Estuarine research: London, Academic Press, p. 217–234.

Breed, C.S., and Grow, T., 1979, Morphology and distribution of dunes in sand seas observed by remote sensing, *in* McKee, E.D., ed., A study of global sand seas: U.S. Geological Survey Professional Paper 1052, p. 253–304.

Breed, C.S., Fryberger, S.G., Andrews, S., McCauley, C., Lennartz, F., Geber, D., and Horstman, K., 1979, Regional studies of sand seas using LANDSAT (ERTS) imagery, *in* McKee, E.D., ed., A study of global sand seas: U.S. Geological Survey Professional Paper 1052, p. 305–398.

Brown, R.A., 1983, The flow in the planetary boundary layer, *in* Brookfield, M.E., and Ahlbrandt, T.S., eds., Eolian sediments and processes: Amsterdam, Elsevier, Developments in Sedimentology 38, p. 291–310.

Folk, R.L., 1976, Rollers and ripples in sand streams and sky: Rhythmic alternation of transverse and longitudinal vortices in three orders: Sedimentology, v. 23, p. 649–669.

Fryberger, S.G., 1979, Dune forms and wind regime, *in* McKee, E.D., ed., A study of global sand seas: U.S. Geological Survey Professional Paper 1052, p. 137–170.

Glennie, K.W., 1970, Desert sedimentary environments: Amsterdam, Elsevier, Developments in Sedimentology 14, 222 p.

Hanna, S.R., 1969, The formation of longitudinal sand dunes by large helical eddies in the atmosphere: Journal of Applied Meteorology, v. 8, p. 874–883.

Havholm, K., and Kocurek, G., 1988, Draa dynamics: A study of a modern draa, Algodones dune field, California: Sedimentology, v. 35 (in press).

Kocurek, G., 1981, Significance of interdune deposits and bounding surfaces in eolian dune sands: Sedimentology, v. 28, p. 753–380.

Lancaster, N., 1983, Controls of dune morphology in the Namib Sand sea, *in* Brookfield, M.E., and Ahlbrandt, T.S., eds., Eolian sediments and processes: Amsterdam, Elsevier, Developments in Sedimentology 38, p. 261–289.

——1985a, Variations in wind velocity and sand transport on the windward flanks of desert dunes: Sedimentology, v. 32, p. 581–593.

——1985b, Winds and sand movements in the Namib sand sea: Earth Surface Processes and Landforms, v. 10, p. 607–619.

——1988, The development of large eolian bed forms: Sedimentary Geology, v. 55, p. 69–89.

Livingstone, I., 1986, Geomorphological significance of wind flow patterns over a Namib linear dune, *in* Nickling, W.G., ed., Aeolian geomorphology: London, Allen and Unwin, p. 97–112.

McKee, E.D., 1979, Introduction to a study of global sand seas, *in* McKee, E.D., ed., A study of global sand seas: U.S. Geological Survey Professional Paper 1052, p. 3–19.

Rubin, D.M., and Hunter, R.E., 1982, Bedform climbing in theory and nature: Sedimentology, v. 29, p. 121–138.

Rubin, D.M., and McCullough, D.S., 1980, Single and superimposed bed forms: A synthesis of San Francisco Bay and flume observations: Sedimentary Geology, v. 26, p. 207–231.

Smith, J.D., and McLean, S.R., 1977, Spatially averaged flow over a wavy surface: Journal of Geophysical Research, v. 82, p. 1735–1746.

Tsoar, H., 1983, Dynamic processes acting on a longitudinal (seif) dune: Sedimentology, v. 30, p. 567–578.

——1986, Two dimensional analysis of dune profiles and the effect of grain size on sand dune morphology, *in* El Baz, F., and Hassan, M.H.A., eds., Physics of desertification: Amsterdam, Martinus Nijhoff, p. 94–108.

Wasson, R.J., and Hyde, R., 1983a, Factors determining desert dune type: Nature, v. 304, p. 337–339.

——1983b, A test of granulometric control of desert dune geometry: Earth Surface Processes and Lndforms, v. 8, p. 301–312.

Wilson, I.G., 1972, Eolian bed forms, their development and origins: Sedimentology, v. 19, p. 173–210.

——1973, Ergs: Sedimentary Geology, v. 10, p. 77–106.

ACKNOWLEDGMENTS

Field work in the Gran Desierto is supported by NASA Ames Research Center Grant NCC 2-346. I thank Ralph Hunter, Gary Kocurek, Dave Rubin, and Steve Wells for their helpful reviews.

Manuscript received October 22, 1987
Revised manuscript received June 29, 1988
Manuscript accepted July 5, 1988

Angles measure compositional differences

G. M. Philip
D. F. Watson
Department of Geology and Geophysics, University of Sydney
Sydney, New South Wales 2006, Australia

ABSTRACT

The geometry of petrological sample space shows that a rock sample, defined by a particular composition or proportion of components, is uniquely described by a radial direction (= compositional axis) from the origin of a Cartesian reference frame where amounts of each component are measured along the axes. The original measured amounts determine the direction of the compositional axis defining a particular mixture of components. Normalization (i.e., expressing amounts as proportions of the whole) merely provides radial projection of the sample onto a proportions frame (three components = the ternary diagram) to permit a form of comparison of different compositions. Such comparisons become inconsistent when traditional parametric approaches are used because, as the "constant sum problem" has long revealed, numbers expressing proportions or percentages are not amenable to standard statistical methods. In addition, because of the disproportionate display of radial difference over the ternary diagram, it cannot be used as a basis for reliable calculations. Recognition of the fundamental radial character of compositional data reveals that they should, most logically, be analyzed on the sphere. Not only does such an approach bypass previously recognized problems in statistical treatment of compositional data, but also it opens the way for the application to petrological data of a whole class of established parametric and nonparametric statistical methods.

INTRODUCTION

Although proportional data (where amounts of components can be expressed as percentages) are common to all branches of geology, nowhere is their analysis of greater importance than in chemical petrology. Here we introduce fundamentals of an approach that bypasses many problems that have arisen with application of traditional statistical methods to such data. Although underlying relations are illustrated using simple numerical examples from petrology, it will be seen that the same approach is applicable to all proportional data.

Data from chemical petrology are presented as parts of the whole (percentages, parts per million, etc.)—i.e., as *proportions*. To calculate the proportion of a component in a mixture, it is necessary to measure the amount of that component and relate it to the total amount of components in the sample (as in a chemical analysis or in point counting). Thus, all such data involve initial measurements of amounts (i.e., magnitudes) which, in turn, can be expressed as proportions.

Sample space of chemical petrology can therefore be cast within the Cartesian coordinate system in orthogonal Euclidean space, the magnitude of each component being measured along its appropriate axis (e.g., Philip et al., 1987, Fig. 1). To make this clear, and to introduce understanding of the geometrical relations developed when amounts are expressed as proportions, initially we discuss two component data.

SAMPLES WITH TWO COMPONENTS

Consider a composition described by two components (such as an aplite consisting of quartz and feldspar). To establish proportions by vol-

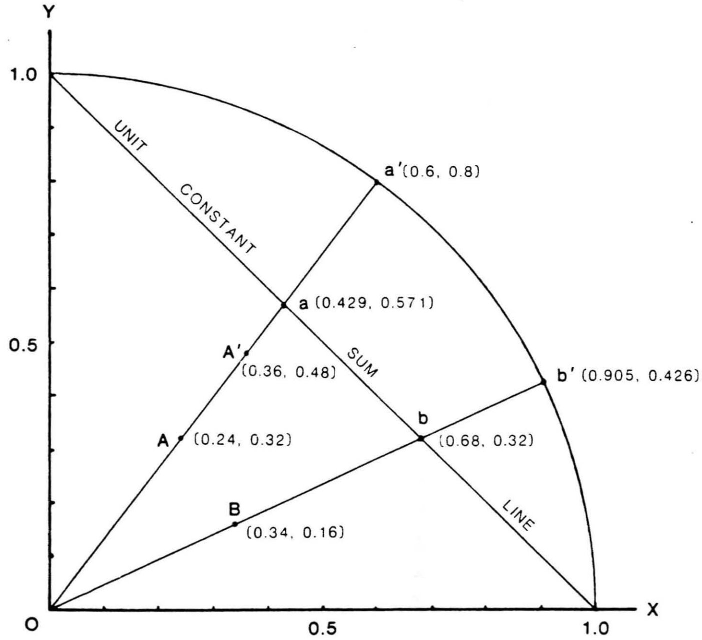

Figure 1. Geometry of petrological sample space for two-component mixtures. Cartesian coordinate system (*X*, *Y*) measures magnitude of individual components contained in sample. Sample A contains 240 *X* and 320 *Y*. Normalizing this to express these amounts as proportions (or percentages) gives coordinates of *a*, radial projection of A to unit constant sum or proportions line. This allows comparison of different samples of proportions. Sample locations can also be projected to unit circle, overcoming radial distortion of angular difference measured along proportions line. See text for further explanation.

ume of the two minerals that compose the rock, point counting is undertaken. Counts total 240 of component X and 320 of component Y, plotting as point A in Figure 1 (in Fig. 1, axial scales and so coordinates are expressed relative to a unit constant-sum line of 1000 to allow direc comparison with Fig. 2). It is immediately apparent that a particula composition marks a direction or, expressed differently, lies on a composi tional axis from the origin along which the ratio of the amounts of X and Y is constant, regardless of the size of the sample analyzed. If the tota number of points counted is increased from 560 to 840, then this particula composition will plot as A′ (ignoring possible counting error). The compo sition is a direction defined by the ratio of the amounts of X and Y; tan a y/x; $a = 53.1301°$ in this example. Of course, the proportions of X and are identical for both point countings, viz. proportion $X = x/(x + y)$ 0.429; proportion $Y = y/(x + y) = 0.571$ (multiplied by 100 to for percentages).